Theoretical physics in transportation

Examination 1, 23. 1. 2015

Abstract

You have 2 hours to answer the questions and prepare the Maple worksheet. I recommend to use comments in your code. As a help you can use your (paper) notebooks, Maple documentation and nothing else.

1 1. Theoretical part

- Define the Lagrangian and quantities entering it.
- Simplify following expression using the summation convention

$$\left((x_1^2 + x_2^2 + x_3^2)\, y_1,\ (x_1^2 + x_2^2 + x_3^2)\, y_2,\ (x_1^2 + x_2^2 + x_3^2)\, y_3\right)$$

- Hamilton's equations
  - What variables does the Lagrangian depend on?
  - What is Legendre transformation?
  - What are the variables in Hamilton's formalism?
  - Derive and write down the Hamilton equations.

2 Maple part

2.1 Rössler attractor

Consider dynamical system

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + a\, y, \\ \dot{z} &= b + z(x - c), \end{aligned} \tag{1}$$

where $x = x(t), y = y(t), z = z(t)$ and a, b, c are real constants. Write down these equations in Maple (with general values a, b, c) together with the initial conditions

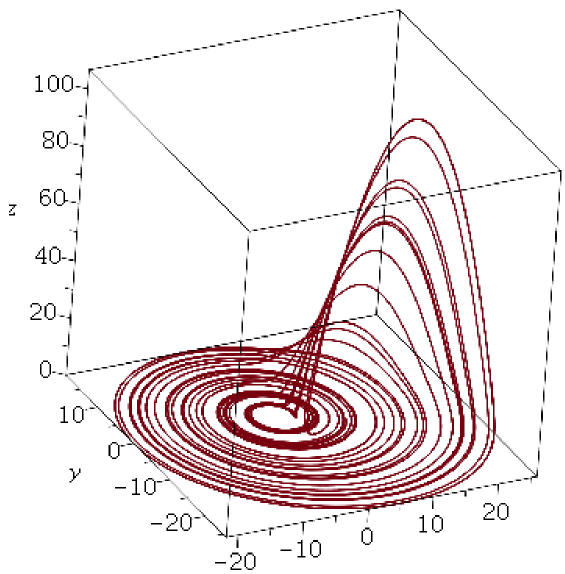
$$x(0) = x_0, \qquad y(0) = y_0, \qquad z(0) = z_0,$$

and store them in the variable called dynsys.

Choose values (using subs)

$$a = b = 0.2, \qquad c = 14, \qquad x_0 = y_0 = 10, \qquad z_0 = 1,$$

and plot the solution. It should look like this:



- Plot the solution shown in the figure.
- How do we call this type of attractor?

2.2 Critical points

Consider dynamical system

$$\begin{aligned} \dot{x} &= x(y+1) - a\,x^2, \\ \dot{y} &= x + y. \end{aligned} \tag{2}$$

- Find the critical points. How many critical points are there? Does the number depend on the value of a?
- Find the Jacobi matrices for all critical points

- Find the eigenvalues of all Jacobi matrices and classify corresponding critical points. If the classification depends on a, perform the analysis and show which classification holds on which interval.
- Plot the solution of system (2) in such a way that for each critical point
  - choose the initial conditions x_0 and y_0 close to the critical point
  - plot the solution
  - if the stability depends on a, plot separate graph for a in each interval
- If the character of the critical point depends on a, plot the bifurcation diagram.
- Are there any values of a for which the critical point is non-hyperbolic? Why is non-hyperbolic point bad?

Hint: if you need to decide where is some function positive and where negative, you can draw the function in Maple and see. Or you can use `solve` to find its zero points.