Theoretical physics in transportation

Examination 2, 30. 1. 2015

Abstract

You have 2 hours to answer the questions and prepare the Maple worksheet. I recommend to use comments in your code. As a help you can use your (paper) notebooks, Maple documentation and nothing else. For each part of the examination you have limited time indicated below. In order to proceed to next parts you must accomplish the tasks at least to 50 %.

1 1. Theoretical part (30 minutes)

- Calculate the force for the potential

$$V(x,y) = -\frac{1}{\sqrt{x^2+y^2}}. \tag{1}$$

- Write down the Lagrangian for a point particle of mass m (in two dimensions) in the potential (1) and convert it to the polar coordinates

$$x = r\,\cos\theta, \qquad y = r\,\sin\theta.$$

- Dynamical systems
  - Definition of $n-$dimensional dynamical system
  - What is autonomous system? What is planar system?
  - Which equations of physics do have the form of dynamical system?

2 Easy Maple part (30 minutes)

- Define the function $f_n(x)$ (of two variables n and x)

$$f_n(x) = \frac{8}{\pi^2}\sum_{k=0}^{n}\frac{(-1)^k}{(2k+1)^2}\sin\left((2k+1)\pi\,x\right)$$

- Plot the function $f_1(x)$ on the interval $x \in (-2\pi, 2\pi)$
- Generate the list of functions (using seq command)

$$\mathsf{functions} = [f_0(x), f_1(x), \dots f_{20}(x)]$$

- Plot the functions stored in this list on the same interval in the single graph.

3 More difficult Maple part (60 minutes)

Consider dynamical system

$$\begin{aligned} \dot{x} &= x(y+1) - a\,x^2, \\ \dot{y} &= x + y. \end{aligned} \tag{2}$$

- Find the critical points. How many critical points are there? Does the number depend on the value of a?
- Find the Jacobi matrices for all critical points
- Find the eigenvalues of all Jacobi matrices and classify corresponding critical points. If the classification depends on a, perform the analysis and show which classification holds on which interval.
- Plot the solution of system (2) in such a way that for each critical point
  - choose the initial conditions x_0 and y_0 close to the critical point
  - plot the solution
  - if the stability depends on a, plot separate graph for a in each interval
- If the character of the critical point depends on a, plot the bifurcation diagram.
- Are there any values of a for which the critical point is non-hyperbolic? Why is non-hyperbolic point bad?

Hint: if you need to decide where is some function positive and where negative, you can draw the function in Maple and see. Or you can use solve to find its zero points.