Minimum for theoretical physics in transportation

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October 11, 2015

Abstract

Series of simple exercises you should be able to solve easily, otherwise you will have problem to follow the lectures. As a **homework**, solve at the tasks in every section. I will collect your homework on October, 16th, or you can send me the solutions in advance to email address `scholtz@utf.mff.cuni.cz`.

1 Derivatives

Find the derivatives of following functions of a single variable.

1. $f(x) = x^2 + \ln x$
2. $f(x) = 2^x$
3. $f(x) = (x^2 + 1)^{10}$
4. $f(x) = \sin x \, \cos x$
5. $f(x) = \cos \omega x$
6. $f(x) = \dfrac{1}{1 + \sin x}$
7. $f(t) = e^{\mathrm{i}\,\omega\,t}$
8. $f(y) = \tan y$
9. $f(x) = \sin(x^2 + 2)$
10. $f(T) = \dfrac{e^{k\,T}}{e^{k\,T} - 1}$

2 Partial derivatives

Find all partial derivatives (i.e. with respect to all variables) of the following functions of several variables.

1. $f(x, y) = x^2 + y^2$
2. $f(x, y) = x^\alpha + y^\beta$
3. $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$
4. $f(x, y, t) = \dfrac{1}{x^2 + y^2} \cos \omega t$
5. $f(x, t) = x^{\alpha t}$
6. $f(\rho, \sigma) = \dfrac{\sigma}{\rho}$
7. $f(\alpha, \beta) = x^\alpha + y^\beta$
8. $f(x, y, t) = e^{\mathrm{i}\,\omega\, t - \mathrm{i}(k_x\, x + k_y\, y)}$
9. $f(x, y) = \cos^2 x$
10. $f(x, y) = 2 \sin x \cos y$

3 Differentials

Recall that whenever you have a function of several variables, say $f = f(x, y)$, the differential of the function is defined by

$$\mathrm{d}f = \frac{\partial f}{\partial x}\,\mathrm{d}x + \frac{\partial f}{\partial y}\,\mathrm{d}y. \tag{1}$$

Of course, this relation generalizes to more variables, e.g.

$$\mathrm{d}f(x, y, z) = \frac{\partial f}{\partial x}\,\mathrm{d}x + \frac{\partial f}{\partial y}\,\mathrm{d}y + \frac{\partial f}{\partial z}\,\mathrm{d}z. \tag{2}$$

Calculate the differentials of following functions (you can use results you obtained in section 2).

1. $f(x, y) = x^2 + y^2$
2. $f(x, y, z, u) = x + y - z + u$
3. $f(x, y) = \sqrt{x^2 + y^2}$
4. $f(x_1, x_2) = \dfrac{x_1}{x_2}$

4 Matrices

1. Find the product $A \cdot B$ of the two matrices given by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3}$$

2. Find the product of matrices

$$A = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}. \tag{4}$$

   How do we call the matrix $A \cdot B$ in this case? What special property does resulting matrix have?

3. Calculate the determinant of matrix

$$R(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \tag{5}$$

   and simplify the result using trigonometric identities.

4. Consider matrix A and vectors u, v, w given by

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad v = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad w = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{6}$$

   (a) Calculate the lengths (norms) of vectors u, v and w and draw the vectors in the plane.

   (b) Calculate vectors

$$u' = A \cdot u, \qquad v' = A \cdot v, \qquad w' = A \cdot w, \tag{7}$$

   and draw new vectors (into separate figure). Also, calculate the lengths of new vectors.

   (c) For what value of ϕ does matrix (5) reduce to matrix A?

   (d) How would you interpret all these results? What is the meaning of matrix $R(\phi)$ given by (5).